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Linear Code Analysis Over $GR(3^2)$ Using Hamming Distance

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Abstract: Data transmission in digital communication systems is vulnerable to disturbances such as noise and interference, which may cause errors in the received information. Therefore, coding mechanisms are required to detect and correct such errors. This study investigates the construction of linear codes over the Galois ring $GR(3^2)$, which is isomorphic to \mathbb{Z}_9 . The code is constructed as a submodule of \mathbb{Z}_9^4 with dimension $k = 2$, meaning that all codewords are formed as linear combinations of two linearly independent generator vectors. Two generator matrices are employed to analyze the effect of generator structure on code performance. All generated codewords are computed and evaluated using Hamming weight and Hamming distance to determine the minimum distance. The results show that the code generated by the first generator matrix has a minimum distance of $d_{\min} = 3$, allowing it to detect up to two errors and correct one error. In contrast, the second generator matrix produces a code with $d_{\min} = 2$, which can only detect a single error without a correction capability. This difference indicates that code performance is more influenced by the linear relationships among generator vectors than by the presence of zero divisors in the ring structure. This study highlights the importance of selecting appropriate generator matrices in constructing linear codes over finite rings and demonstrates the potential of Galois rings as an alternative framework in coding theory.

Keywords: Error Correction, Error Detection, Galois Ring, Hamming Distance, Linear Codes

INTRODUCTION

Reliable data transmission is a fundamental requirement in digital communication systems. Transmitted information is inevitably affected by noise and interference, which can introduce errors in the received data. Therefore, mathematical mechanisms are required to detect and correct such errors (Alabiad & Alkamees, 2021; Jitman & Ling, 2019; Norton & Salagean, 2000). One widely used approach is linear coding, defined as a set of codewords forming a vector subspace over a finite field or a submodule over a finite ring (Huffman & Pless, 2003; MacWilliams & Sloane, 1977). In general, a linear code of length n and dimension k is defined as a k -dimension subspace of \mathbb{F}_q^n , meaning that the code contains q^k codewords generated by k linearly independent



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vectors over a finite field \mathbb{F}_q . When defined over a finite ring such as a Galois ring, the code is viewed as a submodule of R^n (van Lint, 1982; Wood, 1999). This structure enables systematic encoding and analysis using algebraic techniques (Ling & Xing, 2004; Roman, 1992).

Historically, many error-correcting codes have been constructed over finite fields. In particular, binary fields such as $GF(2)$ are used to build well-known codes, including Hamming codes, BCH (Bose–Chaudhuri–Hocquenghem) codes, and Reed-Solomon codes, which are widely applied in communication systems and digital storage (MacWilliams & Sloane, 1977; Pless, 1998). One of the earliest and most influential developments in coding theory was introduced by Hamming, who developed linear codes capable of detecting and correcting single-bit errors (Hamming, 1950).

Although finite fields provide a strong framework for code construction, they have limitations in structural flexibility and symbol representation. Modern communication systems often require coding models capable of representing more general non-binary structures (Dougherty, 2017). As a generalization, finite rings, particularly Galois rings and finite chain rings, offer richer algebraic structures than finite fields (Greferath & Schmidt, 2000; Wan, 2002). This enables the construction of linear codes with greater variation and more complex structural properties (Dinh & López-Permouth, 2004; Gassner et al., 2022; Shuqin & Wenbao, 2004).

Galois rings are particularly important because they can be viewed as extensions of finite fields while preserving certain algebraic properties (Holdman, 2016). Codes constructed over Galois rings have been studied in various forms, such as cyclic and constacyclic codes over $GR(p^k, m)$ (Dinh & López-Permouth, 2004; Sobhani & Esmaeili, 2009; Zhang et al., 2016). Furthermore, several fundamental results from field-based coding theory can be extended to the ring setting (Zhang et al., 2016).

In code analysis, one of the key parameters is the minimum distance, which determines the error detection and correction capability. The Hamming distance is used to measure the difference between codewords and serves as the basis for evaluating code performance (MacWilliams &



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Sloane, 1977; Pless, 1998; van Lint, 1982). In general, a code with minimum distance d_{min} can detect up to $d_{min} - 1$ errors and correct up to $\left\lfloor \frac{d_{min}-1}{2} \right\rfloor$ errors (Lin & Costello, 2004).

This study investigates the construction of linear codes over the Galois ring $GR(3^2)$. The code is constructed through the selection of generator vectors and the analysis of linear independence to form generator matrices. The resulting codewords are then analyzed using Hamming distance to determine the performance parameters of the code. The results are expected to contribute to the development of ring-based coding theory and its applications in digital communication systems.

METHOD

This study employs algebraic and computational approaches to construct and analyze linear codes over the Galois ring $GR(3^2)$. The methods include ring construction, element classification, generator matrix determination, codeword generation, and minimum distance evaluation. The research flow follows general approaches in ring-based coding theory as described in previous studies (Nechaev, 1991; Norton & Salagean, 2000; Wan, 2002).

2.1 Galois Ring Construction

The Galois ring $GR(3^2)$ is a finite commutative ring isomorphic to \mathbb{Z}_9 , consisting of nine elements $\{0,1,2,3,4,5,6,7,8\}$ (Greferath & Schmidt, 2000; Holdman, 2016). Addition and multiplication operations are defined modulo 9 and follow polynomial relations that produce nilpotent elements. In particular, the element x satisfies

$$x^2 \equiv 0 \pmod{9},$$

indicating that the ring contains nilpotent elements and is not a field (Holdman, 2016; Sobhani & Esmacili, 2009; Wan, 2002).



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2.2 Classification of Ring Elements and Its Role in Code Construction

Elements in $GR(3^2) \cong \mathbb{Z}_9$ can be classified based on their multiplicative properties into two main categories: units and zero divisors (Greferath & Schmidt, 1999; Pless, 1998). The set of units is $U = \{1,2,4,5,7,8\}$, consisting of elements with multiplicative inverses in \mathbb{Z}_9 . Meanwhile, the set of zero divisors is $Z = \{0,3,6\}$, where 3 and 6 are nilpotent elements satisfying $x^2 \equiv 0 \pmod{9}$. This classification is used to analyze linear combinations of generator vectors in the formation of codewords. The presence of nilpotent elements allows simultaneous cancellation in certain components, which may produce low-weight codewords and affect the minimum distance (Huffman & Pless, 2003; Pless, 1998).

The linear code is constructed as a submodule of R^n with $R = \mathbb{Z}_9$, using a generator matrix of dimension $k = 2$ and code length $n = 4$. The choice of $k = 2$ enables interaction between two generator vectors, allowing the effects of nilpotent elements in linear combinations to be explicitly observed through possible coordinate-wise cancellations. The code length $n = 4$ provides sufficient positions for such effects to appear without introducing excessive analytical complexity. Thus, the parameter $(n, k) = (4, 2)$ facilitates the identification of the influence of nilpotent elements on the structure of codewords, particularly in generating low-weight codewords and determining the minimum distance of the code.

2.3 Linear Code Construction

Linear codes over rings form submodules rather than vector spaces. Thus, the codes constructed in this study are submodules of $GR(3^2)^n$ (Norton & Salagean, 2000; Wood, 1999). The selection of a generator matrix must satisfy several criteria. The resulting code should form a submodule of R^n , defined as $\mathcal{C} = \{uG \mid u \in R^k\}$, consistent with the definition of linear codes over finite algebraic structures (MacWilliams & Sloane, 1977; van Lint, 1982; Wan, 2002). The rows of the generator matrix must be linearly independent to ensure the desired code dimension (MacWilliams & Sloane, 1977). The matrix must also be free of redundancy so each row contributes independently. Furthermore, pivot elements should be units to ensure invertibility and



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algebraic stability (Wan, 2002). Finally, the generator matrix must produce a non-trivial code with positive dimension and more than one codeword (Ling & Xing, 2004; MacWilliams & Sloane, 1977).

Based on these criteria, two generator matrices are used:

$$G_1 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}.$$

The resulting linear code is defined as $C = \{(a, b)G \mid a, b \in GR(3^2)\}$. Each codeword has the form

$$(a, b)G = a \cdot g_1 + b \cdot g_2.$$

For each generator matrix, the codewords are given by

$$(a, b)G_1 = (a, b, a + 2b, a + b),$$

$$(a, b)G_2 = (a, b, a + 2b, 2a + b).$$

2.4 Minimum Hamming Distance Computation

Let C_1 and C_2 be linear codes generated by G_1 and G_2 , respectively. Each codeword is obtained from linear combinations of generator rows over $GR(3^2)$ (Hamming, 1950; Wan, 2002). The Hamming weight of a codeword $c = (c_1, c_2, \dots, c_n)$ is defined as the number of nonzero components, $w_H(c) = |\{i \mid c_i \neq 0\}|$ (Peterson & Weldon, 1996; Pless, 1998). The Hamming distance between two codewords is defined as $d_H(c_x, c_y) = w_H(c_x - c_y)$ (Blahut, 2003; Hill, 1986). Therefore, the minimum Hamming distance is $d_{min} = \min_{c_x \neq c_y} d_H(c_x, c_y)$, which can be determined by analyzing nonzero codewords. Based on d_{min} , the error detection capability is $d_{min} - 1$ and the error correction capability is $\left\lfloor \frac{d_{min} - 1}{2} \right\rfloor$ (Pless, 1998).

RESULT AND DISCUSSION



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This study investigates the construction of linear codes over the Galois ring $GR(3^2)$, which is isomorphic to \mathbb{Z}_9 . The ring $GR(3^2) \cong \mathbb{Z}_9$ consists of nine elements, namely $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, which serve as the base set for forming the module space used in linear code construction. In this study, the code length is chosen as $n = 4$, resulting in the module space $(GR(3^2))^4 = \mathbb{Z}_9^4$ with a total of $9^4 = 6561$ elements. The constructed linear code is a submodule of dimension $k = 2$, producing $9^2 = 81$ codewords. Based on the Singleton bound, the theoretical minimum distance satisfies $d_{min} \leq n - k + 1 = 3$, indicating that the code can potentially detect up to two errors and correct one error (Blahut, 2003; Huffman & Pless, 2003). However, the actual value of d_{min} must be determined through generator construction, implying that the code performance depends on the structure of the selected generator vectors.

The linear code C is defined as a submodule of \mathbb{Z}_9^4 generated by two linearly independent vectors. Two generator matrices are considered to analyze the impact of generator selection on code structure and performance. For the first generator matrix $g_1 = (1, 0, 1, 1)$ and $g_2 = (0, 1, 2, 1)$, forming

$$G_1 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

For the second generator matrix $g_1 = (1, 0, 1, 2)$ and $g_2 = (0, 1, 2, 1)$, forming

$$G_2 = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

The linear code is constructed through linear combinations of the form

$$C = \{(a \cdot g_1 + b \cdot g_2) \bmod 9 \mid a, b \in \mathbb{Z}_9\}.$$

For example, using the generator matrix G_1 , the codeword corresponding to the message vector $(2, 7)$ is computed as

$$\begin{aligned} C_{(2,7)} &= (2, 7)G_1 = (2 \cdot 1 + 7 \cdot 0, \quad 2 \cdot 0 + 7 \cdot 1, \quad 2 \cdot 1 + 7 \cdot 2, \quad 2 \cdot 1 + 7 \cdot 1) \\ &= (2, 7, 16, 9) \equiv (2, 7, 7, 0) \bmod 9. \end{aligned}$$

A similar procedure is applied to all possible message vectors to generate the complete set of codewords.



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Message	Codeword	Message	Codeword	Message	Codeword
(0,0)	(0,0,0,0)	(3,0)	(3,0,3,3)	(6,0)	(6,0,6,6)
(0,1)	(0,1,2,1)	(3,1)	(3,1,5,4)	(6,1)	(6,1,8,7)
(0,2)	(0,2,4,2)	(3,2)	(3,2,7,5)	(6,2)	(6,2,1,8)
(0,3)	(0,3,6,3)	(3,3)	(3,3,0,6)	(6,3)	(6,3,3,0)
(0,4)	(0,4,8,4)	(3,4)	(3,4,2,7)	(6,4)	(6,4,5,1)
(0,5)	(0,5,1,5)	(3,5)	(3,5,4,8)	(6,5)	(6,5,7,2)
(0,6)	(0,6,3,6)	(3,6)	(3,6,6,0)	(6,6)	(6,6,0,3)
(0,7)	(0,7,5,7)	(3,7)	(3,7,8,1)	(6,7)	(6,7,2,4)
(0,8)	(0,8,7,8)	(3,8)	(3,8,1,2)	(6,8)	(6,8,4,5)
(1,0)	(1,0,1,1)	(4,0)	(4,0,4,4)	(7,0)	(7,0,7,7)
(1,1)	(1,1,3,2)	(4,1)	(4,1,6,5)	(7,1)	(7,1,0,8)
(1,2)	(1,2,5,3)	(4,2)	(4,2,8,6)	(7,2)	(7,2,2,0)
(1,3)	(1,3,7,4)	(4,3)	(4,3,1,7)	(7,3)	(7,3,4,1)
(1,4)	(1,4,0,5)	(4,4)	(4,4,3,8)	(7,4)	(7,4,6,2)
(1,5)	(1,5,2,6)	(4,5)	(4,5,5,0)	(7,5)	(7,5,8,3)
(1,6)	(1,6,4,7)	(4,6)	(4,6,7,1)	(7,6)	(7,6,1,4)
(1,7)	(1,7,6,8)	(4,7)	(4,7,0,2)	(7,7)	(7,7,3,5)
(1,8)	(1,8,8,0)	(4,8)	(4,8,2,3)	(7,8)	(7,8,5,6)
(2,0)	(2,0,2,2)	(5,0)	(5,0,5,5)	(8,0)	(8,0,8,8)
(2,1)	(2,1,4,3)	(5,1)	(5,1,7,6)	(8,1)	(8,1,1,0)
(2,2)	(2,2,6,4)	(5,2)	(5,2,0,7)	(8,2)	(8,2,3,1)
(2,3)	(2,3,8,5)	(5,3)	(5,3,2,8)	(8,3)	(8,3,5,2)
(2,4)	(2,4,1,6)	(5,4)	(5,4,4,0)	(8,4)	(8,4,7,3)
(2,5)	(2,5,3,7)	(5,5)	(5,5,6,1)	(8,5)	(8,5,0,4)
(2,6)	(2,6,5,8)	(5,6)	(5,6,8,2)	(8,6)	(8,6,2,5)
(2,7)	(2,7,7,0)	(5,7)	(5,7,1,3)	(8,7)	(8,7,4,6)
(2,8)	(2,8,0,1)	(5,8)	(5,8,3,4)	(8,8)	(8,8,6,7)



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Table 1. Results of codeword generation using the generator matrix G_1

Message	Codeword	Message	Codeword	Message	Codeword
(0,0)	(0,0,0,0)	(3,0)	(3,0,3,6)	(6,0)	(6,0,6,3)
(0,1)	(0,1,2,1)	(3,1)	(3,1,5,7)	(6,1)	(6,1,8,4)
(0,2)	(0,2,4,2)	(3,2)	(3,2,7,8)	(6,2)	(6,2,1,5)
(0,3)	(0,3,6,3)	(3,3)	(3,3,0,0)	(6,3)	(6,3,3,6)
(0,4)	(0,4,8,4)	(3,4)	(3,4,2,1)	(6,4)	(6,4,5,7)
(0,5)	(0,5,1,5)	(3,5)	(3,5,4,2)	(6,5)	(6,5,7,8)
(0,6)	(0,6,3,6)	(3,6)	(3,6,6,3)	(6,6)	(6,6,0,0)
(0,7)	(0,7,5,7)	(3,7)	(3,7,8,4)	(6,7)	(6,7,2,1)
(0,8)	(0,8,7,8)	(3,8)	(3,8,1,5)	(6,8)	(6,8,4,2)
(1,0)	(1,0,1,2)	(4,0)	(4,0,4,8)	(7,0)	(7,0,7,5)
(1,1)	(1,1,3,3)	(4,1)	(4,1,6,0)	(7,1)	(7,1,0,6)
(1,2)	(1,2,5,4)	(4,2)	(4,2,8,1)	(7,2)	(7,2,2,7)
(1,3)	(1,3,7,5)	(4,3)	(4,3,1,2)	(7,3)	(7,3,4,8)
(1,4)	(1,4,0,6)	(4,4)	(4,4,3,3)	(7,4)	(7,4,6,0)
(1,5)	(1,5,2,7)	(4,5)	(4,5,5,4)	(7,5)	(7,5,8,1)
(1,6)	(1,6,4,8)	(4,6)	(4,6,7,5)	(7,6)	(7,6,1,2)
(1,7)	(1,7,6,0)	(4,7)	(4,7,0,6)	(7,7)	(7,7,3,3)
(1,8)	(1,8,8,1)	(4,8)	(4,8,2,7)	(7,8)	(7,8,5,4)
(2,0)	(2,0,2,4)	(5,0)	(5,0,5,1)	(8,0)	(8,0,8,7)
(2,1)	(2,1,4,5)	(5,1)	(5,1,7,2)	(8,1)	(8,1,1,8)
(2,2)	(2,2,6,6)	(5,2)	(5,2,0,3)	(8,2)	(8,2,3,0)
(2,3)	(2,3,8,7)	(5,3)	(5,3,2,4)	(8,3)	(8,3,5,1)
(2,4)	(2,4,1,8)	(5,4)	(5,4,4,5)	(8,4)	(8,4,7,2)
(2,5)	(2,5,3,0)	(5,5)	(5,5,6,6)	(8,5)	(8,5,0,3)
(2,6)	(2,6,5,1)	(5,6)	(5,6,8,7)	(8,6)	(8,6,2,4)
(2,7)	(2,7,7,2)	(5,7)	(5,7,1,8)	(8,7)	(8,7,4,5)
(2,8)	(2,8,0,3)	(5,8)	(5,8,3,0)	(8,8)	(8,8,6,6)

Table 2. Results of codeword generation using the generator matrix G_1

All generated codewords have been enumerated and presented in Tables 1 and 2. Based on these codewords, Hamming weight and Hamming distance analyses are performed to determine the minimum distance d_{min} .

As an illustration, consider the codewords $c_{(2,7)}$, $c_{(8,4)}$, and $c_{(7,8)}$ generated by G_1 . Their differences are given by

$$c_{(2,7)} - c_{(8,4)} = (2 - 8, 7 - 4, 7 - 7, 0 - 3),$$

$$c_{(2,7)} - c_{(7,8)} = (2 - 7, 7 - 8, 7 - 5, 0 - 6),$$

$$c_{(7,8)} - c_{(8,4)} = (7 - 8, 8 - 4, 5 - 7, 6 - 3).$$

Under arithmetic modulo 9, this becomes

$$c_{(2,7)} - c_{(8,4)} = (3, 3, 0, 6),$$

$$c_{(2,7)} - c_{(7,8)} = (4, 8, 2, 3),$$

$$c_{(7,8)} - c_{(8,4)} = (8, 4, 7, 3).$$

The corresponding Hamming weights are

$$w_H(c_{(2,7)} - c_{(8,4)}) = 3,$$

$$w_H(c_{(2,7)} - c_{(7,8)}) = 4,$$

$$w_H(c_{(7,8)} - c_{(8,4)}) = 4.$$

The minimum Hamming weight for the code generated by G_1 , obtained from all differences $C_{(w,x)} - C_{(y,z)}$ for all codewords is $w_H = 3$, so the minimum Hamming distance is $d_{min} = 3$.

For generator matrix G_2 , consider the same codewords. Their differences are

$$c_{(2,7)} - c_{(8,4)} = (2 - 8, 7 - 4, 7 - 7, 2 - 2),$$

$$c_{(2,7)} - c_{(7,8)} = (2 - 7, 7 - 8, 7 - 5, 2 - 4),$$

$$c_{(7,8)} - c_{(8,4)} = (7 - 8, 8 - 4, 5 - 7, 4 - 2).$$

Under arithmetic modulo 9, this becomes

$$c_{(2,7)} - c_{(8,4)} = (3, 3, 0, 0),$$

$$c_{(2,7)} - c_{(7,8)} = (4, 8, 2, 7),$$

$$c_{(7,8)} - c_{(8,4)} = (8, 4, 7, 2)$$

The corresponding Hamming weights are

$$w_H(c_{(2,7)} - c_{(8,4)}) = 2,$$

$$w_H(c_{(2,7)} - c_{(7,8)}) = 4,$$

$$w_H(c_{(7,8)} - c_{(8,4)}) = 4.$$



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The minimum Hamming weight among all pairwise differences of codewords generated by G_2 is $w_H = 2$. Therefore, the minimum Hamming distance is $d_{min} = 2$.

Based on the Hamming weight and distance analysis from Tables 1 and 2, the code generated by G_1 has $w_H = 3$ and $d_{min} = 3$, while the code generated by G_2 has $w_H = 2$ and $d_{min} = 2$. Therefore, the error detection capability of G_1 is $d_{min} - 1 = 2$, with correction capability $\left\lfloor \frac{d_{min}-1}{2} \right\rfloor = 1$, whereas G_2 can only detect a single error and has no correction capability.

The difference in minimum distance between G_1 and G_2 is not solely due to the presence of zero divisors in the ring \mathbb{Z}_9 , but is more significantly influenced by the linear relationships among the generator vectors. In the case of G_2 , certain linear combinations of the generator vectors produce codewords in which multiple coordinates simultaneously become zero. This is indicated by the existence of nontrivial solutions to a system of equations that force the last two components to vanish. This condition results in codewords with lower Hamming weight, thereby reducing the value of d_{min} . This reduction weakens the codes error detection and corection capability. Thus, although the ring structure includes zero divisors, the dominant factor affecting code performance in this case is the structure of the generator vectors that induce linear dependencies among coordinates. Therefore, the selection of generator vectors is a crucial aspect in determining the quality of linear codes constructed over $GR(3^2)$.

CONCLUSION

The results of this study show that the construction of linear codes over the Galois ring $GR(3^2) \cong \mathbb{Z}_9$ produces linear codes with dimension $k = 2$ and length $n = 4$, with a total of 81 codewords. The analysis shows that the selection of the generator matrix directly affects the minimum Hamming distance and overall code performance. The first generator matrix produces a code with $d_{min} = 3$, enabling detection of up to two errors and correction of one error. In contrast, the second generator matrix yields $d_{min} = 2$, thus limiting the code to single-error detection without correction capability.



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These findings indicate that the reduction in the minimum distance is primarily caused by the presence of linear dependencies among the generator vectors, which result in codewords with low Hamming weight. This confirms that the performance of codes over \mathbb{Z}_9 is not solely determined by the presence of zero divisors, but is more strongly influenced by the structural properties of the generator matrix. Therefore, the selection of generator vectors plays a decisive role in determining the quality of linear codes over Galois rings. This study emphasizes that appropriate generator design is essential, as unsuitable choices may introduce undesirable linear dependencies and degrade code performance, even though Galois ring structures offer greater flexibility than finite fields.

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